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# Graviton propagators on fuzzy $G/H$

**Yoshihisa Kitazawa**

*High Energy Accelerator Research Organization (KEK)*
*Tsukuba, Ibaraki 305-0801, Japan, and*
*Department of Particle and Nuclear Physics*
*The Graduate University for Advanced Studies*
*Tsukuba, Ibaraki 305-0801, Japan*
*E-mail:* kitazawa@post.kek.jp

**Satoshi Nagaoka**

*High Energy Accelerator Research Organization (KEK)*
*Tsukuba, Ibaraki 305-0801, Japan*
*E-mail:* nagaoka@post.kek.jp

Abstract: We describe closed string modes by open Wilson lines in noncommutative (NC) gauge theories on compact fuzzy $G/H$ in IIB matrix model. In this construction the world sheet cut-off is related to the spacetime cut-off since the string bit of the symmetric traced Wilson line carries the minimum momentum on $G/H$. We show that the two point correlation functions of graviton type Wilson lines in 4 dimensional NC gauge theories behave as $1/(\text{momentum})^2$. This result suggests that graviton is localized on D3-brane, so we can naturally interpret D3-branes as our universe. Our result is not limited to D3-brane system, and we generalize our analysis to other dimensions and even to any topology of D-brane worldvolume within fuzzy $G/H$.

Keywords: Gauge-gravity correspondence, D-branes, M(atrix) Theories.

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## Contents



## 1. Introduction

Noncommutative (NC) gauge theory is realized [1−3] by considering a NC background in matrix models [4, 5]. It offers a promising possibility that it contains gravity as a quantum correction through the UV/IR mixing effect [6]. In string theory, some perturbative vacua are well-known and the relation between them are clarified, but there is a vast amount of moduli space to be fixed, and we have no sufficient information to predict which is the nonperturbative vacuum. Landscape is one of the major fields in the recent development in string theory [7]. On the other hand, it is still a very fascinating idea that our universe is uniquely selected through the nonperturbative effect of string theory. To find this mechanism, it is necessary to study quantum gravity from string theory point of view.

Quantum gravity itself is very difficult to study, but in string theory, there is a duality between open string and closed string, therefore, we can analyze quantum gravity by using open string modes. AdS/CFT correspondence is a well-established correspondence [8]. But in an ordinary gauge theory, it might not be easy for us to probe quantum gravity, since we do not keep higher tower of open string degrees of freedom. On the other hand, NC gauge theories may include such open string modes since they are essentially matrix models. In this sense, new effects of quantum gravity might be seen in the quantum corrections of NC gauge theory. Then, what kind of phenomena is included in this effects? One of the possibility which we discuss in this paper is that the 4 dimensional quantum gravity is realized in 4 dimensional NC gauge theory. Our scenario is similar to the brane



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world scenario [9], which explains the localization of gravity on D-brane. We suggest that NC gauge theories provide a localization of gravity on D-branes. Our goal is to derive $1/(\text{momentum})^2$ dependence of massless graviton propagators in NC gauge theories.

In section 2.1, we briefly review the open Wilson lines in NC gauge theories on $S^2 \times S^2$. By considering the regularized space, we can consider the large but finite $N$ system which serves us as a gauge invariant regularization. In section 2.2, two point function of open Wilson lines which couple to the massless graviton mode is calculated. The tensor structure of Wilson line correlators (WLC), which depends on the isometry of $S^2 \times S^2$, is constructed in section 2.3. In section 3.1, we show that the essential part of the correlator, which we explain there in detail, does not depend on our choice of $G/H$. In section 3.2, we calculate two point function of WLC on another homogeneous space, $CP^2$, which has higher symmetry than $S^2 \times S^2$. In section 3.3, we generalize our result to other dimensions, for example, $S^2 \times S^2 \times S^2$. We conclude in section 4 with discussions.

## 2. Wilson line correlators in noncommutative gauge theory

Noncommutative (NC) gauge theories on compact homogeneous spaces can be constructed from IIB matrix model. They have been investigated in [10 − 15]. By considering the compact homogeneous space, we can deal with a large but finite $N$ system, which enables us to investigate non-perturbative questions. It thus serves us as a nonperturbative and gauge invariant regularization of NC gauge theory. The bosonic part of the action of IIB matrix model is written as

$$S = -\frac{1}{4}\,\mathrm{tr}[A_\mu, A_\nu]^2\,, \tag{2.1}$$

where $A_\mu$ are $N \times N$ hermitean matrices and $\mu$ and $\nu$ run over $0, \cdots, 9$. The equation of motion is obtained as

$$[A_\mu, [A_\mu, A_\nu]] = 0\,. \tag{2.2}$$

NC gauge theory is obtained by expanding matrices around the NC backgrounds. We will denote the NC gauge field $a_\mu$ around the background $p_\mu$ as

$$A_\mu = f_\alpha(p_\mu + a_\mu)\,, \tag{2.3}$$

where $f_\alpha$ is a scale factor. When we consider the action (2.1) with a Myers term [16] as

$$\frac{i}{3} f_{\mu\nu\rho} A_\mu [A_\nu, A_\rho]\,, \tag{2.4}$$

we can identify a scale factor $f$ in (2.3) with a coefficient $f$ in (2.4). In this sense, the index $\alpha$ labels the representation of a fuzzy homogeneous space [17]. Alternatively such a space may be realized as a quantum solution [12]. Although supersymmetry is softly broken in either case, the leading behavior of the correlators is constrained by SUSY.



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### 2.1 Feynman rule of noncommutative gauge theory on $S^2 \times S^2$

Let us briefly describe the Feynman rule of NC gauge theory on $S^2$ with $U(1)$ gauge group. We will generalize the rule to $S^2 \times S^2$ background with $U(n)$ gauge group later. We follow the notation in [10].

We expand matrices in terms of matrix spherical harmonics as

$$A^\mu = f_{S^2}\left(p^\mu + \sum_{jm} a^\mu_{jm} Y_{jm}\right), \tag{2.5}$$

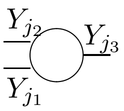
where the representation $Y_{jm}$ is adopted as

$$(Y_{jm})_{ss'} = (-1)^{l-s}\begin{pmatrix} l & j & l \\ -s & m & s' \end{pmatrix}\sqrt{2j+1}. \tag{2.6}$$

$p_\mu$ can be identified with the angular momentum operator in the spin $l$ representation. The normalization is defined as

$$\mathrm{Tr}\, Y_{j_1 m_1} Y_{j_2 m_2} = (-1)^{m_1}\delta_{j_1,j_2}\delta_{m_1,-m_2}\,. \tag{2.7}$$

The cubic vertex of matrix spherical harmonics is written as




$$= \mathrm{Tr}[Y_{j_1 m_1} Y_{j_2 m_2} Y_{j_3 m_3}] = (-1)^{2l}\sqrt{(2j_1+1)(2j_2+1)(2j_3+1)} \\ \times \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}\begin{Bmatrix} j_1 & j_2 & j_3 \\ l & l & l \end{Bmatrix}, \tag{2.8}$$

where we adopt the notation of $(3j)$ and $\{6j\}$ symbols in [18]. The propagators of the NC gauge field $a^\mu_{jm}$ are read from the action as

$$\langle\, a^\mu_{j_1 m_1} a^\nu_{j_2 m_2}\,\rangle = \frac{1}{f^4_{S^2}}\,\frac{(-1)^{m_1}}{j_1(j_1+1)}\,\delta^{\mu\nu}\delta_{j_1 j_2}\delta_{m_1 - m_2}\,. \tag{2.9}$$

Next, let us introduce Wilson lines in NC gauge theory [19] [1] on $S^2$. They are constructed by the trace of polynomial of matrices as

$$y^{\alpha_1,\alpha_2,\cdots,\alpha_j}_{jm}\, Tr A_{\alpha_1}A_{\alpha_2}\cdots A_{\alpha_j}A_{i_1}A_{i_2}\cdots A_{i_k}\,. \tag{2.10}$$

$\alpha = 7, 8, 9$ denote the dimensions where $S^2$ is embedded. $y^{\alpha_1,\alpha_2,\cdots,\alpha_j}_{jm}$ denotes a totally symmetric traceless tensor which corresponds to the spin $j$ representation of $SU(2)$. The background $p_\mu$ consists of angular momentum operators in spin $l$ representation. In our expansion of $A_\mu$ around the background $p_\mu$, the leading term of the Wilson line is written as

$$f^{j+k}_{S^2}\, y^{\alpha_1,\alpha_2,\cdots,\alpha_j}_{jm}\, \mathrm{Tr}\, p_{\alpha_1}p_{\alpha_2}\cdots p_{\alpha_j}\mathcal{O}_1\cdots\mathcal{O}_k\,, \tag{2.11}$$

where $\mathcal{O}$ is a field around the background $p_\mu$. We define $\mathcal{Y}_{jm}$ as

$$\mathcal{Y}_{jm} \equiv y^{\alpha_1,\alpha_2,\cdots,\alpha_j}_{jm}\, p_{\alpha_1}p_{\alpha_2}\cdots p_{\alpha_j}\,. \tag{2.12}$$

[1] The large momentum limit of Wilson line correlators is discussed in [20, 21].

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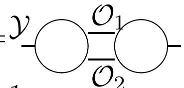
We will focus on the highest weight states of $SU(2)$, therefore, we also define $\mathcal{Y}_j$ as

$$y_{j,j}\,\mathrm{Tr}(p_+)^j\mathcal{O}_1\cdots\mathcal{O}_k = \mathrm{Tr}\,\mathcal{Y}_j\mathcal{O}_1\cdots\mathcal{O}_k\,, \tag{2.13}$$

where $p_+ \equiv p_7 + ip_8$.

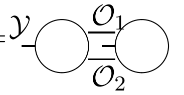
Using these Feynman rules, we find that there are planar and non-planar contribution in the two point function of $\mathrm{Tr}\,\mathcal{Y}_j\mathcal{O}_1\mathcal{O}_2$ at the leading order,

$$\frac{1}{2l+1}\langle \mathrm{Tr}\,\mathcal{Y}_j\mathcal{O}_1\mathcal{O}_2\,\mathrm{Tr}\,\mathcal{O}_2^\dagger\mathcal{O}_1^\dagger\mathcal{Y}_j^\dagger\rangle$$

$=$ 


$$= \langle j|\frac{1}{P_1^2P_2^2}|j\rangle_{\mathrm{p}}\,,$$

$$\frac{1}{2l+1}\langle \mathrm{Tr}\,\mathcal{Y}_j\mathcal{O}_1\mathcal{O}_2\,\mathrm{Tr}\,\mathcal{O}_1^\dagger\mathcal{O}_2^\dagger\mathcal{Y}_j^\dagger\rangle$$

$=$ 


$$= \langle j|\frac{1}{P_1^2P_2^2}|j\rangle_{\mathrm{np}}\,, \tag{2.14}$$

where

$$P_i^\mu\mathcal{Y}_{j_{i'}m_{i'}} \equiv [p^\mu,\mathcal{Y}_{j_{i'}m_{i'}}]\delta_{ii'}\,. \tag{2.15}$$

The planar and nonplanar part of the correlation function on $S^2$ is given by

$$\begin{aligned}\langle j|X|j\rangle_p &= \frac{1}{f_{S^2}^8(2l+1)}\sum_{j_2,j_3,m_2,m_3}\Psi_{123}^*X\Psi_{123}\,,\\ \langle j|X|j\rangle_{np} &= \frac{1}{f_{S^2}^8(2l+1)}\sum_{j_2,j_3,m_2,m_3}\Psi_{132}^*X\Psi_{123}\,,\\ \text{where}\qquad \Psi_{123} &\equiv \mathrm{Tr}\,\mathcal{Y}_{j_3m_3}\mathcal{Y}_{j_2m_2}\mathcal{Y}_j.\end{aligned} \tag{2.16}$$

Now, let us formulate the Wilson line correlators on $S^2\times S^2$ with $U(n)$ gauge group. The construction is the simple extension of the correlators on $S^2$. We expand matrices in terms of the tensor product of matrix spherical harmonics as

$$A_\mu = f_{\mathrm{S}^2\times\mathrm{S}^2}(p_\mu + \sum_{jmpq}a^\mu_{jmpq}Y_{jm}\otimes Y_{pq})\,, \tag{2.17}$$

where

$$\begin{aligned}p_\mu &= j_\mu\otimes 1 \quad (\mu=4,5,6)\,,\\ p_\mu &= 1\otimes\tilde{j}_\mu \quad (\mu=7,8,9)\,.\end{aligned} \tag{2.18}$$

In this section, we consider only the $S^2\times S^2$ manifold, therefore, from now on, we denote $f_{S^2\times S^2}$ as $f$. The summations over $j$ and $p$ run up to $j=2l$ and $p=2l$ respectively. We consider NC gauge theory with $U(n)$ gauge group, so $N=n(2l+1)^2$. The propagators are written as

$$\langle\, a^\mu_{j_1m_1p_1q_1}a^\nu_{j_2m_2p_2q_2}\,\rangle = \frac{1}{f^4}\,\frac{(-1)^{m_1+q_1}}{j_1(j_1+1)+p_1(p_1+1)}\,\delta^{\mu\nu}\delta_{j_1j_2}\delta_{p_1p_2}\delta_{m_1-m_2}\delta_{q_1-q_2}\,. \tag{2.19}$$



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We define the normalization as

$$\operatorname{Tr} Y_{j_1 m_1 p_1 q_1} Y_{j_2 m_2 p_2 q_2} = n(-1)^{m_1} \delta_{j_1 j_2} \delta_{m_1 - m_2} \delta_{p_1 p_2} \delta_{q_1 - q_2} \,. \tag{2.20}$$

The planar and nonplanar part of the correlation function on $S^2 \times S^2$ are given by

$$\begin{aligned}
\langle j,p|X|j,p\rangle_p &= \frac{n^3}{f^8 N} \sum_{j_2,j_3,m_2,m_3} \sum_{p_2,p_3,q_2,q_3} \Psi^*_{123} X \Psi_{123} \,, \\
\langle j,p|X|j,p\rangle_{np} &= \frac{n^3}{f^8 N} \sum_{j_2,j_3,m_2,m_3} \sum_{p_2,p_3,q_2,q_3} \Psi^*_{132} X \Psi_{123} \,, \\
\text{where} \qquad & \Psi_{123} \equiv \operatorname{Tr} \mathcal{Y}_{j_3 m_3 p_3 q_3} \mathcal{Y}_{j_2 m_2 p_2 q_2} \mathcal{Y}_{j,p} \,.
\end{aligned} \tag{2.21}$$

The leading terms of the Wilson lines in the highest weight state representation of $SU(2) \times SU(2)$ are written as

$$y_{j,j} y_{p,p} \operatorname{Tr}(p_+)^j (\tilde{p}_+)^p \mathcal{O}_1 \cdots \mathcal{O}_k = \operatorname{Tr} \mathcal{Y}_{j,p} \mathcal{O}_1 \cdots \mathcal{O}_k \,. \tag{2.22}$$

Finally, we define $\lambda \equiv \frac{n^2}{f^4 N}$, which is identified with 't Hooft coupling.

### 2.2 Two point correlation function of massless graviton mode

The relation between straight Wilson line operators and fields in the massless supergravity multiplet is clarified in [22][23]. In this section, we investigate the two point correlators of a massless graviton mode. The vertex operators which couple to the graviton in type IIB matrix model are written as

$$\begin{aligned}
\operatorname{Str} exp(ik \cdot A) & \left( [A^\rho, A^\mu][A^\rho, A_\nu] + \frac{1}{2} \bar{\psi} \Gamma^{(\nu} [A^{\mu)}, \psi] \right) h_{\nu\mu} \\
& + \frac{1}{2} \operatorname{Str} exp(ik \cdot A) \bar{\psi} \Gamma^{\rho\beta(\nu} \psi [A^{\mu)}, A_\beta] \partial_\rho h_{\nu\mu} \,,
\end{aligned} \tag{2.23}$$

where the symbol Str implies that the ordering of the matrices is defined through the symmetric trace. $(\mu, \nu)$ implies that the Lorentz indices are symmetrized. In analogy with this operator, we may introduce the Wilson line operator in NC gauge theory on $S^2 \times S^2$ as

$$\operatorname{Str} \mathcal{Y}_{j,p}(A) \left( [A_\rho, A_\mu][A_\rho, A_\nu] + \frac{1}{2} \bar{\psi} \Gamma^{(\nu} [A^{\mu)}, \psi] \right) . \tag{2.24}$$

The symmetric trace of the operators on compact space may be defined as

$$\begin{aligned}
\operatorname{Str}(p_+)^j (\tilde{p}_+)^p \mathcal{O}_1 \mathcal{O}_2 \equiv \frac{1}{j} \operatorname{Tr} \sum_{j_1=0}^{j} (p_+)^{j_1} (\tilde{p}_+)^{p_1} \mathcal{O}_1 (p_+)^{j-j_1} (\tilde{p}_+)^{p-p_1} \mathcal{O}_2 \,, \\
\text{where} \quad p_1 \sim \frac{p}{j} j_1 \,,
\end{aligned} \tag{2.25}$$

which is a natural extension of the symmetric trace in the flat noncommutative space. $p_1$ is an integer nearest to $j_1 p/j$. Although supersymmetry is softly broken at the scale where the manifold is curved, it will not affect the leading behavior of the correlators with respect to the large $N$ limit.



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The leading term of the Wilson line is written as

$$\mathrm{Str}\,\mathcal{Y}_{j,p}([p_\rho, a_\mu] - [p_\mu, a_\rho])([p_\rho, a_\nu] - [p_\nu, a_\rho]) \equiv \mathrm{Str}\,\mathcal{Y}_{j,p} f_{\rho\mu} f_{\rho\nu}\,, \tag{2.26}$$

where we define $f_{\rho\mu} \equiv [p_\rho, a_\mu] - [p_\mu, a_\rho]$. Note that there are other terms in the expansion, for example,

$$\mathrm{Str}\,\mathcal{Y}_{j,p}[a_\rho, a_\mu][a_\rho, a_\nu]\,. \tag{2.27}$$

But these terms are of higher orders with respect to the 't Hooft coupling $\lambda$. The two point function of the Wilson line operator which couples to graviton is written as

$$\langle\ \mathrm{Str}\,\mathcal{Y}_{j,p} f_{\rho\mu} f_{\rho\nu}\,\mathrm{Str}\, f^\dagger_{\rho'\mu'} f^\dagger_{\rho'\nu'} \mathcal{Y}^\dagger_{j,p}\ \rangle\,. \tag{2.28}$$

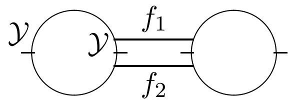
First, we simplify the correlators in such a way that

$$f_{\rho\mu} \to f_1 = [p_\rho, a_\mu], \qquad f_{\rho\nu} \to f_2 = [p_\rho, a_\nu]\,. \tag{2.29}$$

This substitution is useful to understand the essential feature of the correlators. We will present the complete calculation of the correlators in section 2.3.

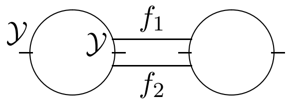
In this way, we obtain

$$\begin{aligned}
&\langle\ \mathrm{Str}\,\mathcal{Y}_{j,p} f_1 f_2\,\mathrm{Str}\, f_2^\dagger f_1^\dagger \mathcal{Y}^\dagger_{j,p}\ \rangle \\
&= y_j^2 y_p^2 \langle\ \mathrm{Str}(p_+)^j (\tilde{p}_+)^p f_1 f_2\,\mathrm{Str}\, f_2^\dagger f_1^\dagger (p_-)^j (\tilde{p}_-)^p\ \rangle \\
&= \frac{y_j^2 y_p^2}{j^2} \sum_{j_1=0}^{j} \sum_{j_2=0}^{j} \langle\ \mathrm{tr}(p_+)^{j_1} (\tilde{p}_+)^{p_1} f_1 (p_+)^{j-j_1} (\tilde{p}_+)^{p-p_1} f_2 \\
&\qquad\qquad \mathrm{tr}\, f_2^\dagger (p_-)^{j-j_2} (\tilde{p}_-)^{p-p_2} f_1^\dagger (p_-)^{j_2} (\tilde{p}_-)^{p_2}\ \rangle \\
&= \frac{y_j^2 y_p^2}{j^2} \sum_{j_1=0}^{j} \sum_{j_2=0}^{j} (y_{j_1} y_{p_1} y_{j_2} y_{p_2} y_{j-j_1} y_{p-p_1} y_{j-j_2} y_{p-p_2})^{-1}
\end{aligned}$$




$$, \tag{2.30}$$

where we denote




$$\equiv \langle\ \mathrm{tr}\,\mathcal{Y}_{j_1,p_1} f_1 \mathcal{Y}_{j-j_1,p-p_1} f_2\,\mathrm{tr}\, f_2^\dagger \mathcal{Y}^\dagger_{j-j_2,p-p_2} f_1^\dagger \mathcal{Y}_{j_2,p_2}\ \rangle\,. \tag{2.31}$$

Before proceeding further, let us show a property of the operator $f_i$, which helps us to perform the calculation:

$$\begin{aligned}
\langle f_1 f_1^\dagger \rangle &\sim \langle [p_\rho, a_\mu][a_\nu^\dagger, p_\rho] \rangle \\
&\sim \sum_{jmpq} \mathcal{Y} P^2 \frac{1}{P^2} (\mathcal{Y}^\dagger) \delta_{\mu\nu} \sim \sum_{jmpq} \mathcal{Y}(\mathcal{Y}^\dagger)\delta_{\mu\nu}\,.
\end{aligned} \tag{2.32}$$

Thus, we can use the completeness condition:

$$\sum_{jmpq} (\mathcal{Y})_{ab} (\mathcal{Y}^\dagger)_{cd} = \delta_{ad}\delta_{bc}\,, \tag{2.33}$$

when we sum over the internal momenta. Here $a, b, c$ and $d$ are indices of matrices. Note that this property does not depend on the choice of the basis.



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Now, let us resume the calculation of (2.30). We substitute the results (2.32) and (2.33) into (2.30) as

$$
\begin{aligned}
&\langle \, \mathrm{Str}\, \mathcal{Y}_{j,p} f_1 f_2 \, \mathrm{Str}\, f_2^\dagger f_1^\dagger \mathcal{Y}_{j,p}^\dagger \, \rangle \\
&= \frac{n^2}{j^2} \sum_{j_1=0}^{j} \sum_{j_2=0}^{j} \frac{y_j^2 y_p^2}{y_{j_1} y_{p_1} y_{j_2} y_{p_2} y_{j-j_1} y_{j-p_1} y_{j-j_2} y_{p-p_2}} \\
&\times \mathrm{tr}\, \mathcal{Y}_{j_1 p_1} \mathcal{Y}_{j_2 p_2}^\dagger \, \mathrm{tr}\, \mathcal{Y}_{j-j_1,p-p_1} \mathcal{Y}_{j-j_2,p-p_2}^\dagger \\
&= \frac{n^2}{j^2} \sum_{j_1=0}^{j} \sum_{j_2=0}^{j} \frac{y_j^2 y_p^2}{y_{j_1} y_{p_1} y_{j_2} y_{p_2} y_{j-j_1} y_{j-p_1} y_{j-j_2} y_{p-p_2}} \delta_{j_1-j_2,0} \delta_{p_1-p_2,0} \\
&= \frac{n^2}{j^2} \sum_{j_1=0}^{j} \Big( \frac{y_j y_p}{y_{j_1} y_{p_1} y_{j-j_1} y_{p-p_1}} \Big)^2 \\
&\equiv \frac{n^2}{j^2} \sum_{j_1=0}^{j} B_{j_1,j-j_1}^2 B_{p_1,p-p_1}^2 \, .
\end{aligned}
\tag{2.34}
$$

where we have introduced the separating function $B_{j_1,j-j_1} = \frac{y_j}{y_{j_1} y_{j-j_1}}$.

$B_{j_1,j-j_1}$ depends on a homogeneous space $G/H$ we consider. As $(p_+)^j = (p_+)^{j_1}(p_+)^{j-j_1}$ leads to $\mathcal{Y}_j = B_{j_1,j-j_1} \mathcal{Y}_{j_1} \mathcal{Y}_{j-j_1}$,

$$
\begin{aligned}
B_{j_1,j-j_1}^{-1} &= \mathrm{tr}\, \mathcal{Y}_j^\dagger \mathcal{Y}_{j_1} \mathcal{Y}_{j-j_1} \\
&= (-1)^{2l} \sqrt{(2j+1)(2j_1+1)(2(j-j_1)+1)} \\
&\times \begin{pmatrix} j & j_1 & j-j_1 \\ j & -j_1 & -j+j_1 \end{pmatrix} \begin{Bmatrix} j & j_1 & j-j_1 \\ l & l & l \end{Bmatrix} ,
\end{aligned}
\tag{2.35}
$$

in the case of $S^2 \times S^2$. $(3j)$ symbol is calculated as

$$
\begin{pmatrix} j & j_1 & j-j_1 \\ j & -j_1 & -j+j_1 \end{pmatrix} = \sqrt{\frac{1}{2j+1}} , \tag{2.36}
$$

while $\{6j\}$ symbol is

$$
\begin{Bmatrix} j & j_1 & j-j_1 \\ l & l & l \end{Bmatrix} \sim \sqrt{\frac{1}{2l}} \begin{pmatrix} j & j_1 & j-j_1 \\ 0 & 0 & 0 \end{pmatrix} , \tag{2.37}
$$

when $l \gg 1$ [24]. Using the Stirling formula $n! \sim \sqrt{2\pi n} n^n e^{-n}$, we obtain

$$
\begin{aligned}
\begin{pmatrix} j & j_1 & j-j_1 \\ 0 & 0 & 0 \end{pmatrix} &= (-1)^j \sqrt{\frac{(2j-2j_1)!(2j_1)!}{(2j+1)!}} \frac{j!}{(j-j_1)! j_1!} \\
&\sim \left( \frac{1}{4\pi j (j-j_1) j_1} \right)^{1/4} .
\end{aligned}
\tag{2.38}
$$

In this way, $B_{j_1,j-j_1}$ is obtained as

$$
B_{j_1,j-j_1}^2 \sim l \sqrt{\frac{\pi j}{j_1 (j-j_1)}} . \tag{2.39}
$$

for $j$, $j_1$, $j - j_1 \gg 1$.



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When the momenta are equally shared: $j = p = K/2$, Wilson line correlator (2.30) is found as

$$\begin{aligned}
&\langle\ \mathrm{Str}\,\mathcal{Y}_{j,p} f_1 f_2\, \mathrm{Str}\, f_2^\dagger f_1^\dagger \mathcal{Y}_{j,p}^\dagger\ \rangle \\
&\sim N \frac{n\pi}{K^2} \log K^2\,.
\end{aligned} \tag{2.40}$$

Thus, we have obtained $1/K^2$ dependence except for the $\log K$ factor. When we consider the correlators with $j \neq p$, they do not exhibit $SO(4)$ symmetry. This undesirable feature may be overcome if we consider the space with higher symmetry. In fact, we will find that there are no log factor nor directional asymmetry in the $CP^2$ space in section 3.2.

### 2.3 Ward identity for Wilson line correlators and tensor structure

In the preceding sub-section, we have found that the graviton two point function behaves as that of a propagator of massless field ($1/K^2$). In this sub-section, we present the complete calculation including the fermionic contribution. We will show that the tensor structure of the Wilson line correlators is consistent with Ward identity.

The two point function of (2.28) is written as

$$\begin{aligned}
&\langle\ \mathrm{Str}\,\mathcal{Y}_{j,p} f_{\rho\mu} f_{\rho\nu}\, \mathrm{Str}\, f_{\rho'\nu'}^\dagger f_{\rho'\mu'}^\dagger \mathcal{Y}_{j,p}^\dagger\ \rangle \\
&= \langle\ \mathrm{Str}\,\mathcal{Y}_{j,p}([p_\rho, a_\mu] - [p_\mu, a_\rho])([p_\rho, a_\nu] - [p_\nu, a_\rho]) \\
&\mathrm{Str}([p_{\rho'}^\dagger, a_{\nu'}^\dagger] - [p_{\nu'}^\dagger, a_{\rho'}^\dagger])([p_{\rho'}^\dagger, a_{\mu'}^\dagger] - [p_{\mu'}^\dagger, a_{\rho'}^\dagger])\mathcal{Y}_{j,p}^\dagger\ \rangle\,,
\end{aligned} \tag{2.41}$$

where we focus on the leading terms of the 't Hooft coupling $\lambda$. Two propagators in this correlator carry almost the same angular momenta since the external angular momentum is assumed to be very small compared to the internal angular momenta of the cut-off scale. It is because the correlator is quartically divergent in power counting. Therefore, we do not distinguish the two propagators and as a result, we obtain the following expression:

$$\begin{aligned}
&\langle\ \mathrm{Str}\,\mathcal{Y}_{j,p}([p_\rho, a_\mu] - [p_\mu, a_\rho])([p_\rho, a_\nu] - [p_\nu, a_\rho]) \\
&\quad \mathrm{Str}([p_{\rho'}^\dagger, a_{\nu'}^\dagger] - [p_{\nu'}^\dagger, a_{\rho'}^\dagger])([p_{\rho'}^\dagger, a_{\mu'}^\dagger] - [p_{\mu'}^\dagger, a_{\rho'}^\dagger])\mathcal{Y}_{j,p}^\dagger\ \rangle \\
&= \frac{1}{K^2} \sum_{K_1, K_2} \sum_{a,b} B_{K_1, K-K_1}^2 B_{K_2, K-K_2}^2 \\
&\mathrm{tr}\mathcal{Y}_1 \mathcal{Y}_a \mathcal{Y}_{1'} \mathcal{Y}_b \left(\frac{1}{P^2}\right)^2 \Big(2(d-2) P^\mu P^{\mu'} P^\nu P^{\nu'} \\
&+ P^2 (2 P^\mu P^\nu \delta_{\mu'\nu'} + 2 P^{\mu'} P^{\nu'} \delta_{\mu\nu} - P^\mu P^{\mu'} \delta_{\nu\nu'} - P^\nu P^{\nu'} \delta_{\mu\mu'} - P^\mu P^{\nu'} \delta_{\mu'\nu} - P^{\mu'} P^\nu \delta_{\mu\nu'}) \\
&+ P^4 (\delta_{\mu\mu'} \delta_{\nu\nu'} + \delta_{\mu\nu'} \delta_{\mu'\nu})\Big)\ \mathrm{tr}\, \mathcal{Y}_b^\dagger \mathcal{Y}_{2'}^\dagger \mathcal{Y}_a^\dagger \mathcal{Y}_2^\dagger\,,
\end{aligned} \tag{2.42}$$

where $d(10)$ is a number of bosonic matrices. $K_1$ and $K_2$ specify the phase structure of the left and right sides of the symmetric trace. $\mathcal{Y}_{i(i')} (i = 1, 2)$ are related to $\mathcal{Y}_{j,p}$ as $\mathcal{Y}_{j,p} = B_{K_i, K-K_i} \mathcal{Y}_i \mathcal{Y}_{i'}$.

On $S^2 \times S^2$, $\sum_{a,b} \operatorname{tr} \mathcal{Y}_1 \mathcal{Y}_a \mathcal{Y}_{1'} \mathcal{Y}_b \operatorname{tr} \mathcal{Y}_b^\dagger \mathcal{Y}_{2'}^\dagger \mathcal{Y}_a^\dagger \mathcal{Y}_2^\dagger = \delta_{j_1-j_2,0}\delta_{p_1-p_2,0}$. We have evaluated the essential part of the correlators in the preceding sub-section as

$$\frac{1}{K^2} \sum_{K_1,K_2} \sum_{a,b} B^2_{K_1,K-K_1} B^2_{K_2,K-K_2} \operatorname{tr} \mathcal{Y}_1 \mathcal{Y}_a \mathcal{Y}_{1'} \mathcal{Y}_b \operatorname{tr} \mathcal{Y}_b^\dagger \mathcal{Y}_{2'}^\dagger \mathcal{Y}_a^\dagger \mathcal{Y}_2^\dagger = \frac{1}{K^2} \sum_{K_1} B^4_{K,K-K1} . \quad (2.43)$$

We will focus on the tensor structure of the correlators in this sub-section.

The leading contribution of the fermionic part of the Wilson line correlators is obtained as

$$\begin{aligned}
&\left\langle \operatorname{Str} \mathcal{Y}_{j,p} \frac{1}{2} \bar{\psi} \Gamma^{(\nu} [p^{\mu)}, \psi] \left( \operatorname{Str} \mathcal{Y}_{j,p} \frac{1}{2} \bar{\psi}' \Gamma^{(\nu'} [p^{\mu')}, \psi'] \right)^\dagger \right\rangle \\
&= \frac{1}{K^2} \sum_{K_1,K_2} \sum_{a,b} B^2_{K_1,K-K_1} B^2_{K_2,K-K_2} \\
&\operatorname{tr} \mathcal{Y}_1 \mathcal{Y}_a \mathcal{Y}_{1'} \mathcal{Y}_b \left( \frac{1}{P^2} \right)^2 \Big( -f P^\mu P^{\mu'} P^\nu P^{\nu'} \\
&+ \frac{f}{8} P^2 (P^\mu P^{\mu'} \delta_{\nu\nu'} + P^\nu P^{\nu'} \delta_{\mu\mu'} + P^\mu P^{\nu'} \delta_{\mu'\nu} + P^{\mu'} P^\nu \delta_{\mu\nu'}) \Big) \operatorname{tr} \mathcal{Y}_b^\dagger \mathcal{Y}_{2'}^\dagger \mathcal{Y}_a^\dagger \mathcal{Y}_2^\dagger , \qquad (2.44)
\end{aligned}$$

where $f(16)$ counts fermionic degrees of freedom. The total amplitude is obtained as

$$\begin{aligned}
A^{\mu\nu\mu'\nu'}_{\text{tot}} =& \frac{1}{K^2} \sum_{K_1,K_2} \sum_{a,b} B^2_{K_1,K-K_1} B^2_{K_2,K-K_2} \\
& \operatorname{tr} \mathcal{Y}_1 \mathcal{Y}_a \mathcal{Y}_{1'} \mathcal{Y}_b (\frac{1}{P^2})^2 \Big( (2d-4-f) P^\mu P^{\mu'} P^\nu P^{\nu'} \\
& - (1 - \frac{f}{8}) P^2 (P^\mu P^{\mu'} \delta_{\nu\nu'} + P^\nu P^{\nu'} \delta_{\mu\mu'} + P^\mu P^{\nu'} \delta_{\mu'\nu} + P^{\mu'} P^\nu \delta_{\mu\nu'}) \\
& + 2P^2 (P^\mu P^\nu \delta_{\mu'\nu'} + P^{\mu'} P^{\nu'} \delta_{\mu\nu}) + P^4 (\delta_{\mu\mu'} \delta_{\nu\nu'} + \delta_{\mu\nu'} \delta_{\mu'\nu}) \Big) \operatorname{tr} \mathcal{Y}_b^\dagger \mathcal{Y}_{2'}^\dagger \mathcal{Y}_a^\dagger \mathcal{Y}_2^\dagger , \quad (2.45)
\end{aligned}$$

In the supersymmetric case ($f = 2(d-2)$), it may be simplified further,

$$\begin{aligned}
A^{\mu\nu\mu'\nu'}_{\text{tot}} =& \frac{1}{K^2} \sum_{K_1,K_2} \sum_{a,b} B^2_{K_1,K-K_1} B^2_{K_2,K-K_2} \\
& \operatorname{tr} \mathcal{Y}_1 \mathcal{Y}_a \mathcal{Y}_{1'} \mathcal{Y}_b \Big( \frac{2}{\tilde{d}} (\tilde{\delta}_{\mu\nu} \delta_{\mu'\nu'} + \delta_{\mu\nu} \tilde{\delta}_{\mu'\nu'}) \\
& + \frac{\frac{d}{4} - \frac{3}{2}}{\tilde{d}} (\tilde{\delta}_{\mu\mu'} \delta_{\nu\nu'} + \tilde{\delta}_{\mu\nu'} \delta_{\mu'\nu} + \delta_{\mu\mu'} \tilde{\delta}_{\nu\nu'} + \delta_{\mu\nu'} \tilde{\delta}_{\mu'\nu}) \\
& + (\delta_{\mu\mu'} \delta_{\nu\nu'} + \delta_{\mu\nu'} \delta_{\mu'\nu}) \Big) \operatorname{tr} \mathcal{Y}_b^\dagger \mathcal{Y}_{2'}^\dagger \mathcal{Y}_a^\dagger \mathcal{Y}_2^\dagger , \qquad (2.46)
\end{aligned}$$

where we have replaced

$$P_{\mu'} P_{\nu'} \to \frac{P^2}{\tilde{d}} \tilde{\delta}_{\mu'\nu'} . \qquad (2.47)$$

$\tilde{d}$ denotes the dimension of the isometry group $G$. $\tilde{\delta}_{\mu'\nu'}$ is a Kronecker delta in the $\tilde{d}$ dimensional subspace.

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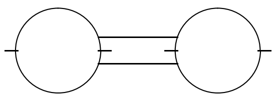
Now, let us consider the tensor structure of graviton correlators on $S^2 \times S^2 = SU(2) \times SU(2)/U(1) \times U(1)$. The dimension of $G = SU(2) \times SU(2)$ is $\tilde{d} = 6$ as they can be embedded in the 6 dimensional space. The total amplitude is obtained from (2.46) as

$$
\begin{aligned}
A_{\rm tot}^{\mu\nu\mu'\nu'} &= \frac{1}{K^2}\sum_{K_1} B^4_{K,K-K1}\Bigg(\frac{1}{3}(\tilde{\delta}_{\mu\nu}\delta_{\mu'\nu'} + \delta_{\mu\nu}\tilde{\delta}_{\mu'\nu'}) \\
&+ \frac{1}{6}(\tilde{\delta}_{\mu\mu'}\delta_{\nu\nu'} + \tilde{\delta}_{\mu\nu'}\delta_{\mu'\nu} + \delta_{\mu\mu'}\tilde{\delta}_{\nu\nu'} + \delta_{\mu\nu'}\tilde{\delta}_{\mu'\nu}) + (\delta_{\mu\mu'}\delta_{\nu\nu'} + \delta_{\mu\nu'}\delta_{\mu'\nu})\Bigg)\,, \qquad (2.48)
\end{aligned}
$$

where we have substituted $f = 16$ and $d = 10$. Tensor structure for the bosonic part is discussed in the appendix.

In order to check the consistency of our calculation, we derive the following Ward identity for the Wilson line correlators of graviton mode

$K_\mu$ (  )

$$
\begin{aligned}
&= K\langle\, \mathrm{Str}\, V^\dagger_{+\nu}\, \mathrm{Str}\, V_{\mu'\nu'}\,\rangle \\
&= \Bigg\langle\, (A_-)^{K+1}_{ij}\left(\frac{\delta}{\delta A_\nu}\right)_{ji} I\,\mathrm{Str}\, V_{\mu'\nu'}\,\Bigg\rangle - \sum_{K_1}\Bigg\langle\, \mathrm{tr}\,\frac{1}{4}(A_-)^{K_1}\bar{\psi}\Gamma_{-\nu}(A_-)^{K-K_1}\frac{\partial}{\partial\bar{\psi}} I\,\mathrm{Str}\, V_{\mu'\nu'}\,\Bigg\rangle \\
&= -\Bigg\langle\, (A_-)^{K+1}_{ij}\left(\frac{\delta}{\delta A_\nu}\right)_{ji} \mathrm{Str}\, V_{\mu'\nu'}\,\Bigg\rangle + \sum_{K_1}\Bigg\langle\, \mathrm{tr}\,\frac{1}{4}(A_-^{K_1}\bar{\psi}\Gamma_{-\nu}(A_-)^{K-K_1}\frac{\partial}{\partial\bar{\psi}}\,\mathrm{Str}\, V_{\mu'\nu'}\,\Bigg\rangle \\
&- \delta_{-\nu}\sum_{K_1}\langle\, \mathrm{tr}(A_-)^{K_1}\,\mathrm{tr}(A_-)^{K-K_1}\,\mathrm{Str}\, V_{\mu'\nu'}\,\rangle\,, \qquad (2.49)
\end{aligned}
$$

where

$$
\begin{aligned}
V_{\mu\nu} &= (A_+)^K\left([A_\rho, A_\mu][A_\rho, A_\nu] + \frac{1}{2}\bar{\psi}\Gamma^{(\mu}[A^{\nu)}, \psi]\right) \\
I &= \frac{1}{4}\,\mathrm{Tr}[A_\mu, A_\rho]^2 + \mathrm{Tr}\,\frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi] \\
(A_\pm)^K &= (p_\pm + a_\pm + \tilde{p}_\pm + \tilde{a}_\pm)^K\,. \qquad (2.50)
\end{aligned}
$$

These vertex operators are closely related to those we have investigated up to a normalization factor of $y_j^2(j!)^2/(2j)!$ since

$$
(p_\pm + \tilde{p}_\pm)^{2j} \sim \frac{(2j)!}{(j!)^2} p^j_\pm \tilde{p}^j_\pm\,. \qquad (2.51)
$$

where $j \gg 1$ is assumed. [2]

First, let us discuss the last line in (2.49). We focus on the leading term of the expansion of 't Hooft coupling $\lambda$. The leading term is one loop diagram. Therefore, the first and second trace can contain no creation (annihilation) operators. Thus, this three point function is calculated as

$$
\sum_{K_1}\langle\, \mathrm{tr}(A_-)^{K_1}\,\mathrm{tr}(A_-)^{K-K_1}\,\mathrm{Str}\, V_{\mu'\nu'}\,\rangle = 0\,, \qquad (2.52)
$$

[2] The two point functions are slightly different since there are no $log(K)$ factors unlike in (2.40).



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since

$$\mathrm{tr}\langle\, (A_-)^K \,\rangle = 0 \qquad \text{for } K \neq 0\,. \tag{2.53}$$

The one point function of Wilson line operators is

$$\left\langle\ -(A_-)^{K+1}_{ij}\left(\frac{\delta}{\delta A_\nu}\right)_{ji}\mathrm{Str}\, V_{\mu'\nu'}\ \right\rangle + \sum_{K_1}\left\langle\ \mathrm{tr}\,\frac{1}{4}(A_-)^{K_1}\bar{\psi}\Gamma_{-\nu}(A_-)^{K-K_1}\frac{\partial}{\partial\bar{\psi}}\mathrm{Str}\, V_{\mu'\nu'}\ \right\rangle . \tag{2.54}$$

The bosonic part is calculated as

$$\begin{aligned}
&(\mathcal{Y}_K)^2\left\langle (A_-)^{K+1}_{ij}\left(\frac{\delta}{\delta A_\nu}\right)_{ji}\mathrm{Str}(A_+)^K\left[A^{\rho'},A^{\mu'}\right]\left[A^{\rho'},A^{\nu'}\right]\right\rangle\\
&=\frac{1}{K}\sum_{K_1}\sum_{a,b}(\mathcal{Y}_K)^2\\
&\mathrm{tr}\,\Big((A_{\rho'}(A_-)^{K+1}(A_+)^{K_1}f^\dagger_{\rho'\nu'}(A_+)^{K-K_1}-A_{\rho'}(A_+)^{K_1}f^\dagger_{\rho'\nu'}(A_+)^{K-K_1}(A_-)^{K+1})\delta_{\nu\mu'}\\
&+(A_{\rho'}(A_-)^{K+1}(A_+)^{K_1}f^\dagger_{\rho'\mu'}(A_+)^{K-K_1}-A_{\rho'}(A_+)^{K_1}f^\dagger_{\rho'\mu'}(A_+)^{K-K_1}(A_-)^{K+1})\delta_{\nu\nu'}\\
&+((A_-)^{K+1}A_{\mu'}(A_+)^{K_1}f^\dagger_{\rho'\nu'}(A_+)^{K-K_1}-A_{\mu'}(A_-)^{K+1}(A_+)^{K_1}f^\dagger_{\rho'\nu'}(A_+)^{K-K_1})\delta_{\rho'\nu}\\
&+((A_-)^{K+1}A_{\nu'}(A_+)^{K_1}f^\dagger_{\rho'\mu'}(A_+)^{K-K_1}-A_{\nu'}(A_-)^{K+1}(A_+)^{K_1}f^\dagger_{\rho'\mu'}(A_+)^{K-K_1})\delta_{\rho'\nu}\Big)\,.
\end{aligned} \tag{2.55}$$

Note that if we consider the noncommutative flat space, there are additional terms which come from the variation of external momenta $e^{ikA}$. We can show that such terms do not contribute to the correlator in this regularization.

The first line of the trace part is calculated as

$$\begin{aligned}
&\frac{1}{K}\sum_{K_1}\sum_{a,b}(\mathcal{Y}_K)^2\\
&\langle\ \mathrm{tr}(A_{\rho'}(A_-)^{K+1}(A_+)^{K_1}f^\dagger_{\rho'\nu'}(A_+)^{K-K_1}-A_{\rho'}(A_+)^{K_1}f^\dagger_{\rho'\nu'}(A_+)^{K-K_1}(A_-)^{K+1})\delta_{\nu\mu'}\ \rangle\\
&=-\frac{1}{K^2}\sum_{K_1,K_2}\sum_{a,b}B_{K_1,K-K_1}B_{K_2,K-K_2}\\
&\quad\mathrm{tr}\,\mathcal{Y}_1\mathcal{Y}_a\mathcal{Y}_{1'}\mathcal{Y}_b\frac{1}{P^2}((d-2)P_{\nu'}K\cdot P+P^2K_{\nu'})\delta_{\nu\mu'}\,\mathrm{tr}\,\mathcal{Y}^\dagger_b\mathcal{Y}^\dagger_{2'}\mathcal{Y}^\dagger_a\mathcal{Y}^\dagger_2\,.
\end{aligned} \tag{2.56}$$

In this way, the bosonic part is obtained as

$$\begin{aligned}
&(\mathcal{Y}_K)^2\left\langle -(A_-)^{K+1}_{ij}\left(\frac{\delta}{\delta A_\nu}\right)_{ji}\mathrm{Str}(A_+)^K[A^{\rho'},A^{\mu'}][A^{\rho'},A^{\nu'}]\right\rangle\\
&\sim\frac{1}{K^2}\sum_{K_1,K_2}\sum_{a,b}B_{K_1,K-K_1}B_{K_2,K-K_2}\,\mathrm{tr}\,\mathcal{Y}_1\mathcal{Y}_a\mathcal{Y}_{1'}\mathcal{Y}_b\\
&\frac{1}{P^2}\Big(((d-2)K\cdot PP_{\nu'}+P^2K_{\nu'})\delta_{\nu\mu'}+((d-2)K\cdot PP_{\mu'}+P^2K_{\mu'})\delta_{\nu\nu'}
\end{aligned}$$



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$$
\begin{aligned}
&- K\cdot P(\delta_{\mu'\nu}P_{\nu'} - P_\nu\delta_{\mu'\nu'}) + P_{\mu'}(P_{\nu'}K_\nu - P_\nu K_{\nu'})\\
&- K\cdot P(\delta_{\nu'\nu}P_{\mu'} - P_\nu\delta_{\nu'\mu'}) + P_{\nu'}(P_{\mu'}K_\nu - P_\nu K_{\mu'})\Big)\,\mathrm{tr}\,\mathcal{Y}_b^\dagger\mathcal{Y}_{2'}^\dagger\mathcal{Y}_a^\dagger\mathcal{Y}_2^\dagger\,. \qquad (2.57)
\end{aligned}
$$

The fermionic part is calculated as

$$
\begin{aligned}
&(\mathcal{Y}_K)^2\left\langle -(A_-)_{ij}^{K+1}\left(\frac{\delta}{\delta A_\nu}\right)_{ji}\mathrm{Str}(A_+)^K\frac{1}{2}\bar\psi\Gamma^{(\nu'}[A^{\mu')},\psi]\right\rangle\\
\to&\frac{1}{K^2}\sum_{K_1,K_2}\sum_{a,b}B_{K_1,K-K_1}B_{K_2,K-K_2}\,\mathrm{tr}\,\mathcal{Y}_1\mathcal{Y}_a\mathcal{Y}_{1'}\mathcal{Y}_b\\
&\frac{f}{4P^2}(P\cdot KP_{\nu'}\delta_{\nu\mu'} + P\cdot KP_{\mu'}\delta_{\nu\nu'})\,\mathrm{tr}\,\mathcal{Y}_b^\dagger\mathcal{Y}_{2'}^\dagger\mathcal{Y}_a^\dagger\mathcal{Y}_2^\dagger\,. \qquad (2.58)
\end{aligned}
$$

The contribution corresponding to fermionic equation of motion is

$$
\begin{aligned}
&(\mathcal{Y}_K)^2\sum_{K_1}\left\langle \mathrm{tr}\,\frac{1}{4}(A_-)^{K_1}\bar\psi\Gamma_{-\nu}(A_-)^{K-K_1}\frac{\partial}{\partial\bar\psi}\mathrm{Str}\,V_{\mu'\nu'}\right\rangle\\
\to&\frac{1}{K^2}\sum_{K_1,K_2}\sum_{a,b}B_{K_1,K-K_1}B_{K_2,K-K_2}\,\mathrm{tr}\,\mathcal{Y}_1\mathcal{Y}_a\mathcal{Y}_{1'}\mathcal{Y}_b\\
&\frac{f}{8P^2}\left((P_\nu(P_{\mu'}K_{\nu'} + P_{\nu'}K_{\mu'}) - P\cdot KP_{\nu'}\delta_{\nu\mu'} - P\cdot KP_{\mu'}\delta_{\nu\nu'}\right)\mathrm{tr}\,\mathcal{Y}_b^\dagger\mathcal{Y}_{2'}^\dagger\mathcal{Y}_a^\dagger\mathcal{Y}_2^\dagger\,. \qquad (2.59)
\end{aligned}
$$

The leading contribution of the one point function of Wilson line operators is given by

$$
\begin{aligned}
&\frac{1}{K^2}\sum_{K_1,K_2}\sum_{a,b}B_{K_1,K-K_1}B_{K_2,K-K_2}\,\mathrm{tr}\,\mathcal{Y}_1\mathcal{Y}_a\mathcal{Y}_{1'}\mathcal{Y}_b\\
&\frac{1}{P^2}\Bigg(\left(d-3-\frac{f}{4}-\frac{f}{8}\right)(P_{\nu'}\delta_{\nu\mu'} + P_{\mu'}\delta_{\nu\nu'})\,K\cdot P + 2P_\nu\delta_{\mu'\nu'}K\cdot P\\
&+(K_{\nu'}\delta_{\nu\mu'} + K_{\mu'}\delta_{\nu\nu'})P^2 + 2K_\nu P_{\mu'}P_{\nu'} + \left(\frac{f}{8}-1\right)P_\nu\left(P_{\mu'}K_{\nu'} + P_{\nu'}K_{\mu'}\right)\Bigg)\,\mathrm{tr}\,\mathcal{Y}_b^\dagger\mathcal{Y}_{2'}^\dagger\mathcal{Y}_a^\dagger\mathcal{Y}_2^\dagger\,.
\end{aligned}
$$
(2.60)

By multiplying $K_\mu$ to $A_{\mathrm{tot}}^{\mu\nu\mu'\nu'}$, we obtain

$$
\begin{aligned}
K_\mu A_{\mathrm{tot}}^{\mu\nu\mu'\nu'} =& \frac{1}{K^2}\sum_{K_1,K_2}\sum_{a,b}B_{K_1,K-K_1}B_{K_2,K-K_2}\,\mathrm{tr}\,\mathcal{Y}_1\mathcal{Y}_a\mathcal{Y}_{1'}\mathcal{Y}_b\\
&\frac{1}{P^2}\Big((2d-4-f)\frac{K\cdot PP^{\mu'}P^\nu P^{\nu'}}{P^2} + 2P_\nu\delta_{\mu'\nu'}K\cdot P + 2K_\nu P_{\mu'}P_{\nu'}\\
&+(\frac{f}{8}-1)((P_{\nu'}\delta_{\nu\mu'} + P_{\mu'}\delta_{\nu\nu'})K\cdot P + P_\nu(P_{\mu'}K_{\nu'} + P_{\nu'}K_{\mu'}))\\
&+P^2(K_{\nu'}\delta_{\nu\mu'} + K_{\mu'}\delta_{\nu\nu'})\Big)\,\mathrm{tr}\,\mathcal{Y}_b^\dagger\mathcal{Y}_{2'}^\dagger\mathcal{Y}_a^\dagger\mathcal{Y}_2^\dagger\,. \qquad (2.61)
\end{aligned}
$$

When $f = 2d - 4$, (2.61) and (2.60) agree with each other.



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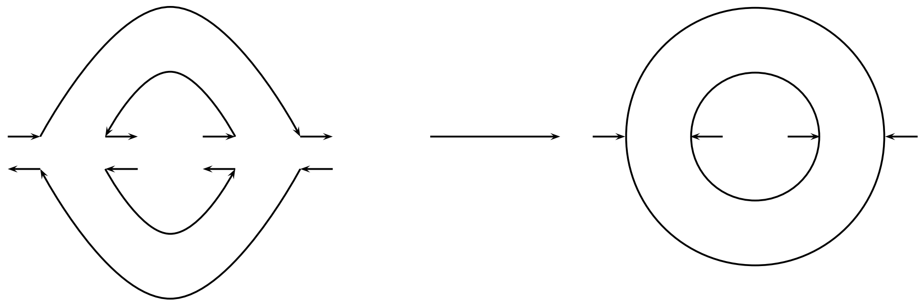


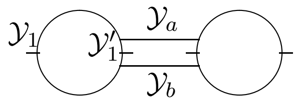
**Figure 1:** 't Hooft's double line notation. We sum over the internal momenta which constitute a complete set of states.

## 3. Universality of the result

### 3.1 Universal amplitude

As we have seen in the previous section, the Wilson line correlator is given by the separating function $B_{j_1,j-j_1}$. In this section, we will show that this result is universal since it only assumes the completeness condition of the generators of $SU(N)$.

The correlators contain the following amplitude




$$= \mathrm{tr}\,\mathcal{Y}_1\mathcal{Y}_a\mathcal{Y}_{1'}\mathcal{Y}_b\,\mathrm{tr}\,\mathcal{Y}_b^\dagger\mathcal{Y}_{2'}^\dagger\mathcal{Y}_a^\dagger\mathcal{Y}_2^\dagger\,. \tag{3.1}$$

We recall the completeness condition:

$$\sum_a (\mathcal{Y}_a)_{ij}(\mathcal{Y}_a^\dagger)_{kl} = \delta_{il}\delta_{jk}\,. \tag{3.2}$$

By using this relation, we obtain

$$\begin{aligned}&\sum_{ab}\mathrm{tr}\,\mathcal{Y}_1\mathcal{Y}_a\mathcal{Y}_{1'}\mathcal{Y}_b\,\mathrm{tr}\,\mathcal{Y}_b^\dagger\mathcal{Y}_{2'}^\dagger\mathcal{Y}_a^\dagger\mathcal{Y}_2^\dagger\\ &= \mathrm{tr}\,\mathcal{Y}_1\mathcal{Y}_2^\dagger\,\mathrm{tr}\,\mathcal{Y}_{1'}\mathcal{Y}_{2'}^\dagger\,.\end{aligned} \tag{3.3}$$

While $\mathcal{Y}$ depends on a particular $G/H$ we pick, the following relation is universal

$$\mathrm{tr}\,\mathcal{Y}_1\mathcal{Y}_2^\dagger = \delta_{j_1-j_2} + \mathcal{O}(1/N)\,, \tag{3.4}$$

where $j_1$ is a momentum carried by $\mathcal{Y}_1$. $\mathrm{tr}\,\mathcal{Y}_{1'}\mathcal{Y}_{2'}^\dagger$ provides the same $\delta$ due to the momentum conservation law. The universality of the amplitude reflects on the universality with respect to the topology of the D-brane worldvolume, which is closely related to the cut off independence of the analysis.

Finally, we provide a pictorial representation of our evaluation of the universal amplitude in figure 1. Our result is naturally understood by using the 't Hooft's double line notation.



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### 3.2 Example : $CP^2$

In contrast to the preceding sub-section, the separating function $B$ depends on a choice of $G/H$. In this sub-section, we will show that the momentum ($k$) dependence of WLC on $CP^2 = SU(3)/U(2)$ is also as $1/k^2$. We will calculate $B$ in the semiclassical approximation. We define the raising and lowering operators as

$$p_\pm = \frac{1}{\sqrt{2}}(p_4 \pm i p_5)\,, \qquad \tilde{p}_\pm = \frac{1}{\sqrt{2}}(p_6 \pm i p_7)\,, \tag{3.5}$$

The normalization condition of spherical harmonics is

$$\operatorname{tr} \mathcal{Y}_j^\dagger \mathcal{Y}_j = 1\,, \tag{3.6}$$

where

$$\mathcal{Y}_j = y_j (p_+)^j\,. \tag{3.7}$$

In the semiclassical approximation,

$$p_+ = r\frac{\xi_1}{1+\bar{\xi}\xi}\,, \qquad p_- = r\frac{\xi_2}{1+\bar{\xi}\xi}\,, \tag{3.8}$$

we may estimate

$$\begin{aligned} \operatorname{tr} \mathcal{Y}_j^\dagger \mathcal{Y}_j &= r^{2j+2} \int \frac{2d^4\xi}{\pi^2(1+\xi\bar{\xi})^3} \frac{(\bar{\xi}\xi)^j}{(1+\bar{\xi}\xi)^{2j}} y_j^2 \\ &= r^{2j+2} \frac{2(j!)^2}{(2j+2)!} y_j^2\,. \end{aligned} \tag{3.9}$$

Thus, we obtain

$$\begin{aligned} \tilde{B}^2_{j_1, j-j_1} &= \frac{y_j^2}{y_{j-j_1}^2 y_{j_1}^2} \\ &\sim \frac{\sqrt{\pi}}{2} \left( \frac{j}{(j-j_1)j_1} \right)^{\frac{3}{2}} N\,. \end{aligned} \tag{3.10}$$

The Wilson line correlators (2.30) are calculated as

$$\begin{aligned} &\langle\, \operatorname{Str} \mathcal{Y}_j f_1 f_2 \operatorname{Str} f_2^\dagger f_1^\dagger \mathcal{Y}_j^\dagger \,\rangle \\ &= \frac{1}{j^2} \sum_{j_1=0}^{j} \sum_{j_2=0}^{j} \langle\, \operatorname{tr} \mathcal{Y}_{j_1} f_1 \mathcal{Y}_{j-j_1} f_2 \operatorname{tr} f_2^\dagger \mathcal{Y}_{j_2}^\dagger f_1^\dagger \mathcal{Y}_{j-j_2}^\dagger \rangle B_{j_1 j-j_1} B_{j_2 j-j_2} \\ &\sim \frac{N}{j^2} \sqrt{\pi} \zeta\left(\frac{3}{2}\right)\,. \end{aligned} \tag{3.11}$$

We have obtained the $1/(\text{momentum})^2$ behavior without a log factor. The correlators are also invariant under the rotation of 8 dimensional space in which $CP^2$ sits.



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### 3.3 Universality with respect to the dimensionality

We have shown in this section that the correlator is given by the separating function $B$. This result holds for any $G/H$ , irrespective of its dimension. Therefore, we consider higher dimensional NC gauge theory here. NC gauge theory on $S^2 \times S^2 \times S^2$ is considered in [14]. The WLC is obtained as

$$\begin{aligned}
&\left\langle \,\mathrm{Str}(p_{a+})^j(p_{b+})^j(p_{c+})^j f_1 f_2 \,\mathrm{Str}\, f_2^\dagger f_1^\dagger (p_{a-})^j(p_{b-})^j(p_{c-})^j \right\rangle \\
&= \frac{n^2}{j^2}\sum_{j_1=0}^{j}\left(\frac{y_j}{y_{j_1}y_{j-j_1}}\right)^6 \\
&= \frac{n^2}{j^2}\sum_{j_1=0}^{j} B^6_{j_1,j-j_1} \\
&\sim \frac{n^2}{j^2}\sum_{j_1=1}^{j}\left(\frac{l^2\pi j}{j_1(j-j_1)}\right)^{\frac{3}{2}} \\
&= \frac{Nn\pi^{3/2}}{4j^2}\zeta(\frac{3}{2})\,. \qquad (3.12)
\end{aligned}$$

Thus, the graviton is localized on 6 dimensional subspace: $S^2 \times S^2 \times S^2$. We may naturally interpret that graviton is localized on D5-brane.

When we consider $(S^2\times)^x$ type spacetime, correlators are calculated as

$$\frac{n^2}{j^2}\sum B^{2x} \sim N\frac{n}{j^2}\,. \qquad (3.13)$$

except $S^2(x=1)$. Thus, the correlators exhibit the inverse squared momentum law on any $G/H$ whose dimension is larger than 2.

## 4. Conclusions and discussions

In this paper, we have investigated the two point correlation functions of graviton vertex operators in 4 dimensional NC gauge theory with maximal SUSY on compact homogeneous spacetime $G/H$. The infrared contributions ($k^4 log(k)$) to the correlators are identical to those in conformal field theory just like the correlators of the energy-momentum tensor. However the ultra-violet contributions are very different even in the small external momentum case. This is due to the UV/IR mixing effects caused by the NC phases in the correlators. In the case of the symmetric ordered graviton operators, we find that the two point correlators behave as $1/k^2$. This fact indicates the existence of massless gravitons in NC gauge theory. It has been clear that there is a bulk gravity in 4d NC gauge theory with maximal SUSY since the one loop effective action involving the quadratic Wilson lines is consistent with 10 dimensional supergravity. In order to obtain realistic quantum gravity, we need to obtain 4 dimensional gravity. Such a possibility may be realized in various ways if a graviton is bound to the brane or through induced gravity on the brane. We hope



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our findings will make a first concrete step to identify such a mechanism in 4d NC gauge theory.

We still need to investigate various issues to establish such a mechanism. One issue is to understand the correlators of $n$ point functions. Another issue is to understand the correlators of more generic Wilson lines. If we consider the vertex operators which contain more commutators of $[A_\mu, A_\nu]$, analogous calculations show that the two point functions are more singular in the infra-red limit than $1/k^2$. It might imply that the relevant modes are (gravitationally) confined and develop a mass gap in that channel. On the other hand, the correlators of the Wilson lines which contain fewer $[A_\mu, A_\nu]$ do not exhibit singularity in the infra-red limit. The third issue is that the two point correlators are not transverse due to the one point functions as we have seen in the Ward identity. They seem to correspond to graviton propagators in a certain gauge.

Our investigation is also restricted to the leading order of the 't Hooft coupling in NC gauge theory which is valid in the weak coupling regime. We need to understand higher order quantum corrections also. Since the behavior of the correlators is governed by the power counting, it is likely that higher order corrections do not modify our results. It is also desirable to have a consistent supergravity description in the strong coupling limit.

If the graviton vertex operators are coupled to conserved energy-momentum tensor, we can reproduce the Newton's law between them by taking the expectation values of the graviton vertex operators. It might be a good strategy to pursue this idea further since such a structure is consistent with the one loop effective action of NC gauge theory.


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## A. Bosonic part of the tensor structure of graviton correlators on $S^2 \times S^2$

In this appendix, we investigate the bosonic part of the tensor structure of graviton correlators on $S^2 \times S^2$. We obtain the anisotropic tensor structure. For the supersymmetric correlators, we obtain the isotropic tensor structure in section 2.3. By considering the isometry of the space, we can replace

$$\begin{aligned}
P_\mu P_\nu P'_\mu P'_\nu \to \frac{P_A^4}{15}&(\delta_A^{\mu\nu}\delta_A^{\mu'\nu'} + \delta_A^{\mu\mu'}\delta_A^{\nu\nu'} + \delta_A^{\mu\nu'}\delta_A^{\mu'\nu}) \\
&+ \frac{P_A^2 P_B^2}{9}(\delta_A^{\mu\nu}\delta_B^{\mu'\nu'} + \delta_A^{\mu\mu'}\delta_B^{\nu\nu'} + \delta_A^{\mu\nu'}\delta_B^{\mu'\nu}) \\
&+ \frac{P_A^2 P_B^2}{9}(\delta_B^{\mu\nu}\delta_A^{\mu'\nu'} + \delta_B^{\mu\mu'}\delta_A^{\nu\nu'} + \delta_B^{\mu\nu'}\delta_A^{\mu'\nu}) \\
&+ \frac{P_B^4}{15}(\delta_B^{\mu\nu}\delta_B^{\mu'\nu'} + \delta_B^{\mu\mu'}\delta_B^{\nu\nu'} + \delta_B^{\mu\nu'}\delta_B^{\mu'\nu})\,, \qquad \text{(A.1)}
\end{aligned}$$



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where $\delta_A$ and $\delta_B$ are Kronecker delta effective to the 3 dimensions,

$$\delta_A^{\mu\nu} = \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{pmatrix}, \delta_B^{\mu\nu} = \begin{pmatrix} 0 & & & & & \\ & 0 & & & & \\ & & 0 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{pmatrix}$$

$$P_A^2 = P_4^2 + P_5^2 + P_6^2, \quad P_B^2 = P_7^2 + P_8^2 + P_9^2 \,. \tag{A.2}$$

By using (A.1), the bosonic part of the correlator (2.42) is replaced as

$$\begin{aligned} &\left(\frac{16}{15}P_A^4 + \frac{2}{3}P^4\right)(\delta_A^{\mu\nu}\delta_A^{\mu'\nu'} + \delta_A^{\mu\mu'}\delta_A^{\nu\nu'} + \delta_A^{\mu\nu'}\delta_A^{\mu'\nu}) \\ &+\left(\frac{16}{9}P_A^2P_B^2 + \frac{2}{3}P^4\right)(\delta_A^{\mu\nu}\delta_B^{\mu'\nu'} + \delta_A^{\mu\mu'}\delta_B^{\nu\nu'} + \delta_A^{\mu\nu'}\delta_B^{\mu'\nu}) \\ &+\left(\frac{16}{9}P_A^2P_B^2 + \frac{2}{3}P\right)(\delta_A^{\mu\nu}\delta_B^{\mu'\nu'} + \delta_A^{\mu\mu'}\delta_B^{\nu\nu'} + \delta_A^{\mu\nu'}\delta_B^{\mu'\nu}) \\ &+\left(\frac{16}{15}P_B^4 + \frac{2}{3}P^4\right)(\delta_B^{\mu\nu}\delta_B^{\mu'\nu'} + \delta_B^{\mu\mu'}\delta_B^{\nu\nu'} + \delta_B^{\mu\nu'}\delta_B^{\mu'\nu}) \,. \end{aligned} \tag{A.3}$$

We need to estimate the $P_A^4$ and $P_A^2P_B^2$. We calculate them under the semiclassical approximation. Angular momenta are represented by the adjoint representation on $S^2$, then, the integral of $P_A^4$ is semiclassically written as

$$\int dX_1 d\tilde{X}_1 \frac{(X_1 - X_2)^4}{(X_1 - X_2)^2 + (\tilde{X}_1 - \tilde{X}_2{}^2)} \tag{A.4}$$

where

$$P_A = X_1 - X_2 \,. \tag{A.5}$$

$X_2$ and $\tilde{X}_2$ are fixed at some point on $S^2$. (A.4) is calculated as

$$\begin{aligned} &\int d\Omega d\tilde{\Omega} \frac{(X_1 - X_2)^4}{(X_1 - X_2)^2 + (\tilde{X}_1 - \tilde{X}_2{}^2)} \\ &= \int_0^\pi d\cos\theta d\cos\tilde{\theta} \frac{(2 - 2\cos^2\theta)^2}{(4 - 2\cos^2\theta - 2\cos^2\tilde{\theta})^2} \\ &= \int_{-1}^1 dX d\tilde{X} \frac{(1 - X^2)^2}{(2 - X^2 - \tilde{X}^2)^2} \,, \end{aligned} \tag{A.6}$$

where we transform the valuables as

$$X = \cos\theta, \qquad \tilde{X} = \cos\tilde{\theta} \,. \tag{A.7}$$

The integral of $P_A^2P_B^2$ is also estimated as

$$\int_{-1}^1 dX d\tilde{X} \frac{(1 - X^2)(2 - \tilde{X}^2)}{(2 - X^2 - \tilde{X}^2)^2} \,. \tag{A.8}$$



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By carrying out the integration of $\tilde{X}$ in (A.8), we obtain

$$4\int_0^1 dX(-1+X^2)\left(-\frac{-1+X^2}{2(-2+X^2)(-1+X^2)}+\frac{(-3+X^2)\tan^{-1}\frac{1}{\sqrt{-2+X^2}}}{2(-2+X^2)^{3/2}}\right). \tag{A.9}$$

The first term is calculated as

$$-2+\sqrt{2}\log(1+\sqrt{2})\,. \tag{A.10}$$

The second term is calculated as

$$-4\int_1^{\sqrt{2}} dx\frac{(x^2+1)(x^2-1)}{2x^2\sqrt{2-x^2}}\tanh^{-1}\frac{1}{x}\,, \tag{A.11}$$

where we transform the valuables as

$$X^2-2=-x^2\,. \tag{A.12}$$

Formally, $\tanh(1/x)$ is expanded as

$$\tanh^{-1}\frac{1}{x}=\sum_{k=1}^{\infty}\frac{1}{2k-1}\left(\frac{1}{x}\right)^{2k-1}\,. \tag{A.13}$$

By using this expression, we carry out the integral in (A.11) as

$$\sum_{k=1}^{\infty}\frac{-4}{2k-1}\left(-\frac{2^{-5/2-k}}{k(k-2)}(\frac{(k-2)\sqrt{\pi}\Gamma(1-k)}{\Gamma(1/2-k)}-\frac{4k\sqrt{\pi}\Gamma(3-k)}{\Gamma(5/2-k)})\right.$$
$$\left.+\frac{-(k-2){}_2F_1((1/2,1),(1-k),-1)+k{}_2F_1((1/2,1),(3-k),-1)}{4k(k-2)}\right), \tag{A.14}$$

where ${}_2F_1(a;b;z)$ is a generalized hypergeometric function. We numerically obtain (A.8) as

$$\begin{aligned}\int_0^1 dXd\tilde{X}\frac{(1-X^2)(2-\tilde{X}^2)}{(2-X^2-\tilde{X}^2)^2} &\sim -0.188+0.396\\ &=0.208\,.\end{aligned} \tag{A.15}$$

We also evaluate (A.6) as

$$\int_0^1 dXd\tilde{X}\frac{(1-X^2)^2}{(2-X^2-\tilde{X}^2)^2}\sim 0.292\,. \tag{A.16}$$

After all, the Bosonic part of the tensor structure of the graviton on $S^2\times S^2$ (A.3) is evaluated among the estimations (A.15) and (A.16) as

$$\begin{aligned}&0.98(\delta_A^{\mu\nu}\delta_A^{\mu'\nu'}+\delta_A^{\mu\mu'}\delta_A^{\nu\nu'}+\delta_A^{\mu\nu'}\delta_A^{\mu'\nu}+\delta_B^{\mu\nu}\delta_B^{\mu'\nu'}+\delta_B^{\mu\mu'}\delta_B^{\nu\nu'}+\delta_B^{\mu\nu'}\delta_B^{\mu'\nu})\\ &+1.04(\delta_A^{\mu\nu}\delta_B^{\mu'\nu'}+\delta_A^{\mu\mu'}\delta_B^{\nu\nu'}+\delta_A^{\mu\nu'}\delta_B^{\mu'\nu}+\delta_A^{\mu\nu}\delta_B^{\mu'\nu'}+\delta_A^{\mu\mu'}\delta_B^{\nu\nu'}+\delta_A^{\mu\nu'}\delta_B^{\mu'\nu})\,.\end{aligned} \tag{A.17}$$



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